

Shifted focus point scenario from the minimal mixed mediation of SUSY breaking

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We employ both the minimal gravity- and the minimal gauge mediations of supersymmetry breaking at the grand unified theory (GUT) scale in a single supergravity framework, assuming the gaugino masses are generated dominantly by the minimal gauge mediation effects [1]. In such a “minimal mixed mediation model,” a “focus point” of the soft Higgs mass parameter, $m_{h_u}^2$ emerges at 3-4 TeV energy scale, which is exactly the stop mass scale needed for explaining the 126 GeV Higgs boson mass without the “A-term” at the three loop level. As a result, $m_{h_u}^2$ can be quite insensitive to various trial stop masses at low energy, reducing the fine-tuning measures to be much smaller than 100 even for a 3-4 TeV low energy stop mass and $-0.5 < A_t/m_0 \lesssim +0.1$ at the GUT scale. The gluino mass is predicted to be about 1.7 TeV, which could readily be tested at LHC run2.

Although the minimal supersymmetric standard model (MSSM) has been believed the most promising theory beyond the standard model (SM), guiding the SM to a grand unified theory (GUT) or string theory [2, 3], any evidence of SUSY has not been observed yet at the large hadron collider (LHC). The mass bounds on the SUSY particles have gradually increased, and now they seem to start threatening the traditional status of SUSY as a prominent solution to the naturalness problem of the SM. Actually, a barometer of the naturalness of the MSSM is the mass of “stop.” Due to the large top quark Yukawa coupling (y_t), the top and stop dominantly contribute to the radiative physical Higgs mass squared and also the renormalization of a soft mass squared of the Higgs ($m_{h_u}^2$) in the MSSM. The renormalization effect on $m_{h_u}^2$ would *linearly be sensitive* to the stop mass squared, while it depends just logarithmically on a ultraviolet (UV) cutoff [2]. Since the Higgs mass parameters, $m_{h_u}^2$ and $m_{h_d}^2$ are related to the Z boson mass m_Z together with the “Higgsinos” mass, μ [2],

$$\frac{1}{2}m_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2, \quad (1)$$

$\{m_{h_u}^2, m_{h_d}^2, |\mu|^2\}$ should be finely tuned to yield $m_Z^2 = (91 \text{ GeV})^2$ for a given $\tan \beta [\equiv \langle h_u \rangle / \langle h_d \rangle]$, if they are excessively large. According to the recent analysis based on the three-loop calculations, the stop mass required for explaining the 126 GeV Higgs boson mass [4] without any other helps is about 3-4 TeV [5]. Thus, a fine-tuning of order 10^{-3} or smaller looks unavoidable in the MSSM for a GUT scale cut-off.

In order to more clearly see the UV dependence of $m_{h_u}^2$ and properly discuss this “little hierarchy problem”, however, one should suppose a specific UV model and analyze its resulting full renormalization group (RG) equations. One nice idea is the “focus point (FP) scenario” [6]. It is based on the minimal gravity mediation (mGrM) of SUSY breaking. So the soft mass squareds such as $m_{h_{u,d}}^2$ and those of the left handed (LH) and right handed (RH) stops, $(m_{q_3}^2, m_{u_3^c}^2)$ as well as the gaugino masses M_a ($a = 3, 2, 1$) are given to be *universal* at the GUT scale, $m_{h_u}^2 = m_{h_d}^2 = m_{q_3}^2 = m_{u_3^c}^2 = \dots \equiv m_0^2$ and $M_3 = M_2 = M_1 \equiv m_{1/2}$. As pointed out in [6], if the soft SUSY breaking “A-terms” are zero at the GUT scale and the unified gaugino mass $m_{1/2}$ is just a few hundred GeV, $m_{h_u}^2$ converges to a small negative value around the Z boson mass scale in this setup, *regardless of its initial values given by m_0^2 at the GUT scale* [6]. In the RG solution of $m_{h_u}^2$ at the m_Z scale, thus,

$$m_{h_u}^2(Q = m_Z) = C_s m_0^2 - C_g m_{1/2}^2, \quad (2)$$

where C_s, C_g (> 0) can numerically be estimated using RG equations, C_s happens to be quite small with the above universal soft masses. Since stop masses are quite sensitive to m_0^2 , hence, m_Z^2 could remain small enough even with a relatively heavy stop mass in the FP scenario in contrast to the naive expectation.

However, the experimental bound on the gluino mass M_3 has already exceeded 1.3 TeV [7]. As expected from Eqs. (1) and (2), a too large $m_{1/2}$ needed for $M_3 > 1.3 \text{ TeV}$ at low energy would require a fine-tuned large $|\mu|$ for m_Z of 91 GeV particularly for a relatively light stop mass ($\lesssim 1 \text{ TeV}$) cases. When the stop mass is around 3-4 TeV, the stop should decouple from the RG equations below 3-4 TeV, which makes C_s *sizable* in Eq. (2) [8]. Then, a much larger $m_{1/2}$ is necessary for EW symmetry breaking. Since the RG running interval between 3-4 TeV and m_Z scale, to which modified RG equations should be applied, is too large, the FP behavior is seriously spoiled with such heavy SUSY particles.

The best way to rescue the FP idea is to somehow shift the FP upto the stop decoupling scale [8]: C_s needs to be made small enough before stops are decoupled. Then $m_{h_u}^2$ at the m_Z scale can be estimated using the

Coleman-Weinberg potential [2, 9]. It is approximately given by

$$m_{h_u}^2(m_Z) \approx m_{h_u}^2(Q_T) - \frac{3|y_t|^2}{16\pi^2} (m_{q_3}^2 + m_{u_3^c}^2) \Big|_{Q_T}, \quad (3)$$

where Q_T denotes the stop decoupling scale. Since the m_0^2 dependence of stop masses would be loop-suppressed, $m_{h_u}^2$ needs to be well-focused around Q_T . Due to the additional negative contribution to $m_{h_u}^2(m_Z)$ below Q_T , a small *positive* $m_{h_u}^2(Q_T)$ would be more desirable. In order to push up the FP to the desired stop mass scale 3-4 TeV, we suggest to combine the mGrM and the minimal gauge mediation (mGgM) in a single supergravity (SUGRA) framework with a *common* SUSY breaking source. We will call it “minimal mixed mediation.”

First, let us consider the minimal Kähler potential, and a superpotential where the observable and hidden sectors are separated as in the ordinary mGrM [2]:

$$K = \sum_{i,a} |z_i|^2 + |\phi_a|^2, \quad W = W_H(z_i) + W_O(\phi_a) \quad (4)$$

where z_i [ϕ_a] denotes fields in the hidden [observable] sector. The kinetic terms of z_i and ϕ_a , thus, take the canonical form. We assume non-zero vacuum expectation values (VEVs) for z_i s [3]:

$$\langle z_i \rangle = b_i M_P, \quad \langle \partial_{z_i} W_H \rangle = a_i^* m M_P, \quad \langle W_H \rangle = m M_P^2, \quad (5)$$

where a_i and b_i are dimensionless numbers, while M_P ($\approx 2.4 \times 10^{18}$ GeV) is the reduced Planck mass. Then, $\langle W_H \rangle$ or m gives the gravitino mass, $m_{3/2} = e^{K/2M_P} \langle W \rangle / M_P^2 = e^{|b_i|^2/2} m$, and the “ F -terms” of z_i ($= D_{z_i} W = \partial_{z_i} W + \partial_{z_i} K W / M_P^2$) become of order $\mathcal{O}(m M_P)$. The soft terms can read from the scalar potential of SUGRA: when the cosmological constant (C.C.) is fine-tuned to be zero, renormalizable terms of it are given by [3]

$$V_F \approx |\partial_{\phi_a} W_O|^2 + m_0^2 |\phi_a|^2 + m_0 [\phi_a \partial_{\phi_a} W_O + (A_\Sigma - 3) W_O + \text{h.c.}], \quad (6)$$

where A_Σ is defined as $A_\Sigma \equiv \sum_i b_i^* (a_i + b_i)$, and m_0 is identified with the gravitino mass $m_{3/2}$ ($= e^{|b_i|^2/2} m$). The first term of Eq. (6) is the F -term potential in global SUSY, the second term is the universal soft mass term, and the remaining terms are A -terms, which are *proportional* to m_0 .

Next, let us introduce one pair of messenger superfields $\{\mathbf{5}, \bar{\mathbf{5}}\}$, which are the SU(5) fundamental representations. Through their coupling with a SUSY breaking source S , which is an MSSM singlet superfield,

$$W_m = y_S S \mathbf{5} \bar{\mathbf{5}}, \quad (7)$$

the soft masses of the MSSM gauginos and scalar superpartners are also radiatively generated [2]:

$$M_a = \frac{g_a^2}{16\pi^2} \frac{\langle F_S \rangle}{\langle S \rangle}, \quad m_i^2 = 2 \sum_{a=1}^3 \left[\frac{g_a^2}{16\pi^2} \frac{\langle F_S \rangle}{\langle S \rangle} \right]^2 C_a(i) \quad (8)$$

where $C_a(i)$ is the quadratic Casimir invariant for a superfield i , $(T^a T^a)_i^j = C_a(i) \delta_i^j$, and g_a ($a = 3, 2, 1$) denotes the MSSM gauge couplings. $\langle S \rangle$ and $\langle F_S \rangle$ are VEVs of the scalar and F -term components of the superfield S . The mGgM effects would appear below the messenger scale, $y_S \langle S \rangle$. Here we assume that $\langle S \rangle$ has the same magnitude as the VEV of the SU(5) breaking Higgs v_G : $\langle \mathbf{24}_H \rangle = v_G \times \text{diag.}(2, 2, 2; -3, -3)/\sqrt{60}$. It is possible if a GUT breaking mechanism causes $\langle S \rangle$. Actually, the “ X ” and “ Y ” gauge boson masses, $M_X^2 = M_Y^2 = \frac{5}{24} g_G^2 v_G^2$ [10], where g_G is the unified gauge coupling, can be identified with the MSSM gauge coupling unification scale.

In addition to Eq. (4), the Kähler potential (and hidden local symmetries we don’t specify here) can permit

$$K \supset f(z) S + \text{h.c.}, \quad (9)$$

where $f(z)$ denotes a *holomorphic* monomial of hidden sector fields z_i s with VEVs of order M_P in Eq. (5), and so it is of order $\mathcal{O}(M_P)$. Their kinetic terms still remain canonical. The $U(1)_R$ symmetry forbids $M_P f(z) S$ in the superpotential. Then, the resulting $\langle F_S \rangle$ can be $\langle F_S \rangle \approx m [\langle f(z) \rangle + \langle S^* \rangle]$ by including the SUGRA corrections with $\langle W_H \rangle = m M_P^2$. Thus, the VEV of F_S is of order $\mathcal{O}(m M_P)$ like F_{z_i} . They should be fine-tuned for the vanishing C.C.: a precise determination of $\langle F_S \rangle$ is indeed associated with the C.C. problem. Here we set $\langle F_S \rangle = m_0 M_P$. Thus, the typical size of mGgM effects is estimated as

$$\frac{\langle F_S \rangle}{16\pi^2 \langle S \rangle} = \frac{m_0 M_P}{16\pi^2 M_X} \sqrt{\frac{5}{24}} g_G \approx 0.36 \times m_0, \quad (10)$$

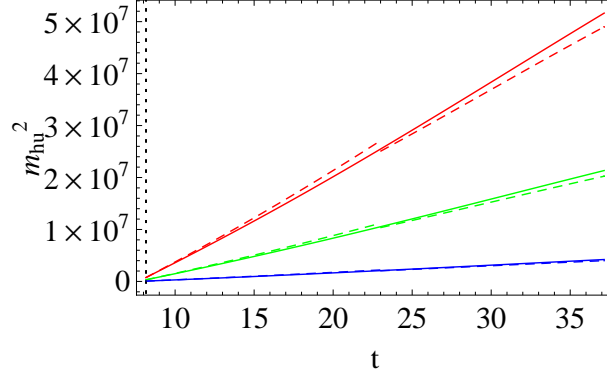


FIG. 1: RG evolutions of $m_{h_u}^2$ with $t \equiv \log(Q/\text{GeV})$ for $m_0^2 = (7\text{ TeV})^2$ [Red], $(4.5\text{ TeV})^2$ [Green], and $(2\text{ TeV})^2$ [Blue] when $A_t = -0.2 m_0$ and $\tan\beta = 50$. The tilted straight [dotted] lines correspond to the case of $t_M \approx 37$ (or $Q_M \approx 1.3 \times 10^{16}$ GeV, “Case A”) [$t_M \approx 23$ (or $Q_M = 1.0 \times 10^{10}$ GeV, “Case B”)]. The vertical dotted line at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) indicates the desired stop decoupling scale. The discontinuities of $m_{h_u}^2(t)$ should appear at the messenger scales. As seen in the figure, the FP scale is not affected by messenger scales.

where g_G is set to be $\sqrt{4\pi/26}$ at the GUT scale [$\approx (1.3 \pm 0.4) \times 10^{16}$ GeV] due to relatively heavy colored superpartners ($\gtrsim 3$ TeV). Even for $|y_S| \ll 1$, we will keep this value, since it is fixed by a UV model. For $|y_S| \lesssim 1$ in Eq. (7), the messenger scale Q_M drops down below $M_{X,Y}$. The soft masses generated by the mGgM in Eq. (8) are *non-universal* for $Q_M < M_{X,Y}$, and the beta function coefficients of the MSSM fields should be modified above the Q_M scale by the messenger fields $\{5, \bar{5}\}$. The boundary conditions at the GUT scale are of the universal form as seen in Eq. (6). We have *additional* non-universal contributions by Eq. (8). They should be imposed at a given messenger scale, and so affect the RG evolutions of MSSM parameters for $Q \leq Q_M$.

We also suppose that the gaugino masses from the mGrM are relatively suppressed. In fact, the gaugino mass term in SUGRA is associated with the first derivative of the gauge kinetic function [3], and so a constant gauge kinetic function at tree level ($= \delta_{ab}$) can realize it. Thus, the gaugino masses by Eq. (8) dominates over them in this case. Then, a simple analytic expression for the gaugino masses at the stop mass scale is possible: $M_a(Q_T) \approx 0.36 \times m_0 \times g_a^2(Q_T)$. It does *not* depend on messenger scales, A_t , $\tan\beta$, etc.

The fact that the mGgM effects by Eq. (8) are proportional to m_0 or m_0^2 are important. Moreover, A -terms from Eq. (6) are also proportional to m_0 . In this setup, thus, an (extrapolated) FP of $m_{h_u}^2$ must still exist at a higher energy scale. As C_g is converted to a member of C_s in Eq. (2), the naturalness of $m_{h_u}^2$ and m_Z^2 becomes gradually improved, making C_s smaller and smaller, until the FP reaches the stop decoupling scale.

Fig. 1 displays RG evolutions of $m_{h_u}^2$ under various trial m_0^2 s. The straight [dotted] lines correspond to the case of $t_M \approx 37$ (or $Q_M \approx 1.3 \times 10^{16}$ GeV, “Case A”) [$t_M \approx 23$ (or $Q_M = 1.0 \times 10^{10}$ GeV, “Case B”)]. The discontinuities of the lines by additional boundary conditions arise at the messenger scales. As seen in Fig. 1, a FP of $m_{h_u}^2$ appears always at $t = t_T \approx 8.2$ (or $Q_T \approx 3.5$ TeV) *regardless of the chosen messenger scales*. Hence, the wide ranges of UV parameters can yield almost the same values of $m_{h_u}^2$ at low energy. Under this situation, one can guess that $m_0^2 \approx (4.5\text{ TeV})^2$ happens to be selected, yielding 3-4 TeV stop mass, and so eventually gets responsible for the 126 GeV Higgs mass. In both cases of Fig. 1, the low energy gaugino masses are

$$M_{3,2,1} \approx \{1.7\text{ TeV}, 660\text{ GeV}, 360\text{ GeV}\} \quad (11)$$

for $m_0^2 = (4.5\text{ TeV})^2$. They would be testable at LHC run2. A_t at low energy is about 1 TeV for Case A and B. So the contributions of A_t^2/\tilde{m}_t^2 to the radiative Higgs mass are smaller than 2.3 % of those by the stops.

Table I lists the soft squared masses at $t = t_T$ for the LH and RH stops, and the two MSSM Higgs bosons under the various m_0^2 s, when $Q_M \approx 1.3 \times 10^{16}$ GeV, and $\tan\beta$ is 50 or 25. We can see the changes of $m_{h_u}^2$ are quite small [$\ll (550\text{ GeV})^2$] under the changes of m_0^2 [$(5.5\text{ TeV})^2 - (3.5\text{ TeV})^2$], because $m_{h_u}^2$ is well-focused at $t = t_T$. Case I-IV yield again the same low energy gauginos masses as Eq. (11). A_t at low energy turns out to be around 1 TeV or smaller for $m_0^2 = (4.5\text{ TeV})^2$, and so its contribution to the Higgs boson mass is still suppressed. By Eq. (3) $m_{h_u}^2$ s further decrease to be negative below $t = t_T$. With Eq. (1) $|\mu|$ are determined as $\{485\text{ GeV}, 392\text{ GeV}, 516\text{ GeV}, 586\text{ GeV}\}$ for Case I, II, III, and IV, respectively. In Table I, the fine-tuning measure $\Delta_{m_0^2}$ ($\equiv \left| \frac{\partial \log m_Z^2}{\partial \log m_0^2} \right| = \left| \frac{m_0^2}{m_Z^2} \frac{\partial m_Z^2}{\partial m_0^2} \right|$ [11]) around $m_0^2 = (4.5\text{ TeV})^2$ are also presented. They are of order $\mathcal{O}(1 - 10)$. Δ_{A_t} ($\equiv \left| \frac{A_t}{m_Z^2} \frac{\partial m_Z^2}{\partial A_t} \right|$) is estimated as $\{0, 10, 118, 0\}$ for Case I, II, III, and IV, respectively. When

TABLE I: Soft squared masses of the stops and Higgs bosons at $t = t_T \approx 8.2$ ($Q_T = 3.5$ TeV) for various trial m_0^2 s when $Q_M \approx 1.3 \times 10^{16}$ GeV. $\Delta_{m_0^2}$ indicates the fine-tuning measure for $m_0^2 = (4.5 \text{ TeV})^2$ in each case. $m_{h_u}^2$ s further decrease to be negative below $t = t_T$. The mass spectra are generated using SOFTSUSY [12].

Case I	$A_t = 0$	$\tan \beta = 50$	$\Delta_{m_0^2} = 1$	Case II	$A_t = -0.2 m_0$	$\tan \beta = 50$	$\Delta_{m_0^2} = 16$
m_0^2	(5.5 TeV) ²	(4.5 TeV) ²	(3.5 TeV) ²	m_0^2	(5.5 TeV) ²	(4.5 TeV) ²	(3.5 TeV) ²
$m_{q_3}^2(t_T)$	(4363 GeV) ²	(3551 GeV) ²	(2744 GeV) ²	$m_{q_3}^2(t_T)$	(4376 GeV) ²	(3563 GeV) ²	(2752 GeV) ²
$m_{u_3^c}^2(t_T)$	(3789 GeV) ²	(3098 GeV) ²	(2406 GeV) ²	$m_{u_3^c}^2(t_T)$	(3798 GeV) ²	(3106 GeV) ²	(2413 GeV) ²
$m_{h_u}^2(t_T)$	(431 GeV) ²	(189 GeV) ²	-(251 GeV) ²	$m_{h_u}^2(t_T)$	(539 GeV) ²	(361 GeV) ²	-(44 GeV) ²
$m_{h_d}^2(t_T)$	(2022 GeV) ²	(1512 GeV) ²	(1008 GeV) ²	$m_{h_d}^2(t_T)$	(2053 GeV) ²	(1565 GeV) ²	(1046 GeV) ²
Case III	$A_t = -0.5 m_0$	$\tan \beta = 50$	$\Delta_{m_0^2} = 9$	Case IV	$A_t = 0$	$\tan \beta = 25$	$\Delta_{m_0^2} = 57$
m_0^2	(5.5 TeV) ²	(4.5 TeV) ²	(3.5 TeV) ²	m_0^2	(5.5 TeV) ²	(4.5 TeV) ²	(3.5 TeV) ²
$m_{q_3}^2(t_T)$	(4284 GeV) ²	(3532 GeV) ²	(2630 GeV) ²	$m_{q_3}^2(t_T)$	(4915 GeV) ²	(4025 GeV) ²	(3134 GeV) ²
$m_{u_3^c}^2(t_T)$	(3755 GeV) ²	(3088 GeV) ²	(2373 GeV) ²	$m_{u_3^c}^2(t_T)$	(3770 GeV) ²	(3086 GeV) ²	(2400 GeV) ²
$m_{h_u}^2(t_T)$	-(363 GeV) ²	-(41 GeV) ²	-(546 GeV) ²	$m_{h_u}^2(t_T)$	(152 GeV) ²	-(220 GeV) ²	-(293 GeV) ²
$m_{h_d}^2(t_T)$	(1447 GeV) ²	(1359 GeV) ²	-(950 GeV) ²	$m_{h_d}^2(t_T)$	(5057 GeV) ²	(4136 GeV) ²	(3215 GeV) ²

$A_t/m_0 = +0.1$, $\{\Delta_{m_0^2}, \Delta_{A_t}, |\mu|\}$ turn out to be about $\{22, 33, 569 \text{ GeV}\}$. Therefore, the parameter range

$$-0.5 < A_t/m_0 \lesssim +0.1 \quad \text{and} \quad \tan \beta \gtrsim 25 \quad (12)$$

allows $\{\Delta_{m_0^2}, \Delta_{A_t}\}$ and $|\mu|$ to be smaller than 100 and 600 GeV, respectively [1]. We see that a larger $\tan \beta$ would be preferred for a smaller $\Delta_{m_0^2}$. It is basically because $m_{h_d}^2$ is not focused unlike $m_{h_u}^2$, even if it also contributes to m_Z^2 as seen in Eq. (1). $\tan \beta = 50$ is easily obtained e.g. from the minimal SO(10) GUT [10].

In conclusion, we have noticed that a FP of $m_{h_u}^2$ appears at 3-4 TeV, when the mGrM and mGgM effects are combined at the GUT scale for a common SUSY breaking source parametrized with m_0 , and the gaugino masses are dominantly generated by the mGgM effects. Even for a 3-4 TeV stop mass explaining the 126 GeV Higgs mass, thus, the fine-tuning measures significantly decrease well below 100 for $-0.5 < A_t/m_0 \lesssim +0.1$ and $\tan \beta \gtrsim 25$ in the minimal mixed mediation. In this range, $|\mu|$ is smaller than 600 GeV. The expected gluino mass is about 1.7 TeV, which could readily be tested at LHC run2.

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